FORMATION OF SHOCK WAVES BY AN ELECTRIC DISCHARGE IN WATER

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When an electric discharge takes place in a liquid, the discharge channel rapidly expands, a process acompanied by the radiation of a compression wave. If the rate of expansion of the channel approaches the speed of sound, a shock wave is formed at the leading front of the radiated compression pulse.

The characteristics of the channel-expansion process and the parameters of the shock wave can be determined from the given conditions of energy release in the discharge channel. For simplicity we will consider a discharge for which the width of the discharge gap is small as compared with the characteristic radius of the channel, which makes it possible to assume that the channel is spherical. The motion of the liquid caused by the expansion of the channel will be assumed isentropic, which makes it possible to use the equation of state in the form:

$$p = A\,(\rho / \rho_0)^n - B\,, \tag{1}$$

where ρ is the liquid density, p is the pressure, $A = 3001$ atm $B = 3000$ atm $n = 7$ for water.

In calculating the hydrodynamic characteristics of the discharge it is possible to distinguish two successively solvable problems: 1) calculation of the expansion of the channel for a given regime of energy release, and 2) determination of the shock-wave radiation by a channel expanding according to a known law.

The channel expansion process can be approximately described by the system of equations

$$P\frac{dV}{d\tau} + \frac{1}{\gamma - 1}\frac{d(PV)}{d\tau} = N(\tau) \qquad (V = 4\pi R^3/3)\,, \tag{2}$$

$$R\frac{d^2R}{d\tau^2}\left(1 - \frac{U}{c}\right) + \frac{3}{2}\left(1 - \frac{U}{3c}\right)U^2 = \left(1 + \frac{U}{c}\right)H + \frac{R}{c}\left(1 - \frac{U}{c}\right)\frac{dH}{d\tau}\,, \tag{3}$$

$$H = \int\limits_{p_0}^{p}\frac{dp}{\rho} = \frac{c^2 - c_0^2}{n - 1} = \frac{c_0^2}{n - 1}\left[\left(\frac{P + B}{A}\right)^{(n-1)/n} - 1\right] \tag{4}$$

The first equation expresses the law of conservation of energy in the discharge [1, 2], while the second and third may be regarded as relations giving the pressure at the surface of the expanding sphere, equal to the pressure inside channel P, as a function of the channel radius R and its derivative $R^{\cdot} = U$ and $R^{\cdot}$.

Equations (3) and (4) follow from the hydrodynamic equations and the equation of state when the approximations of the Kirkwood-Bethe theory [3, 4], proposed for describing the propagation of shock waves in a liquid, are employed.

The remaining notation is as follows: V is the channel volume, γ is the effective adiabatic exponent for plasma; for discharges in water [1] $\gamma = 1.2$, $N(\tau)$ is the power released in the channel, c is the local speed of sound, H is the enthalpy. The initial conditions are selected as follows $R \to 0, R^{\cdot} \to 0$ as $\tau \to 0$. In partice it is sufficient to assign values of R and U at $\tau = 0$ sufficiently small when compared to the characteristic values of those quantities.

Numerical intergration of (2)–(4) makes it possible to determine the time dependence of the channel radius, the rate of expansion, the pressure in the channel, and the values of the function $G(\tau) = R(H + U^2/2)$ on the surface of the expanding sphere. The determination of this function —the starting point in the solution of the propagation problem in the Kirkwood-Bethe theory—presupposes that the values of the function $G = r(h + u^2/2)$ (r is the radial coordinate, h the specific enthalpy, and u is the hydrodynamic velocity at the point r) remain constant at points traveling with velocity c + u, which makes it possible to determine this function at any point in space from the known values of G at (τ) at the surface of the sphere. In practice it is more convenient to calculate the inverse of the function G(tr) from the equations [4]

$$t(G, r) = \tau(G) + \frac{G\beta}{c_0^2}\left[\frac{1+2\beta u}{\beta u(1+\beta u)} - \frac{1+\beta U}{\beta U(1+\beta U)} - 2\ln\frac{(1+\beta u)\beta U}{\beta u(1+\beta U)}\right],$$
$$\left(U = u(R, \tau),\ \ \beta u = \frac{1}{2}\left[\left(1 + \frac{n+1}{rc_0^2}G\right)^{1/2} - 1\right],\ \ \beta = \frac{n+1}{4c_0}\right). \tag{5}$$

In small perturbations ($\beta u \ll 1$) (5) goes over into the solution $t - (r - R)/c_0 = \tau(G)$ corresponding to the approximation of linear acoustics.

From the known values of the function $G(t, r)$ it is easy to determine the hydrodynamic velocity and the pressure p [4],

$$p = A\left[\frac{2}{n+1} + \frac{n-1}{n+1}\left(1 + \frac{n+1}{rc_0^2}G\right)^{1/2}\right]^{2n/(n-1)} - B, \tag{6}$$

and easy to find the profile of the compression wave at any point in space.

At a sufficient distance from the discharge the profile of the compression wave may become multivalued, which, as is known [5], indicates the formation of shock fronts. Their position and the magnitude of the discontinuity are determined from the solution obtained using the Rankine–Hugoniot relation, which in weak shocks reduces to the simple "equal area" rule [5].

At considerable distances from the discharge, where nonlinear effects lead to intense distortion of the original wave profile, the shock wave takes a form that depends only slightly on the detailed properties of the function $G(\tau)$ at the channel surface. This makes it possible to obtain simple asymptotic expression describing shock wave remote from the discharge which, in particular, reveal the nature of its attenuation. Two cases are possible. If the rate of expansion of the channel is equal to or exceed the speed of sound, shock waves develop in the immediate vicinity of the discharge. In this case it is possible to use the asymptotic expressions of the theory developed by Kirkwood and Bethe for describing the shock waves due to an explosion and applicable at a considerable distance from the explosion center [3]. In accordance with this theory, the shape of the wave is assumed approximately exponential:

$$p = p_m \exp\frac{-t}{\theta},\quad p_m = \frac{\rho G_m}{r},\quad G_m = xG_0,$$
$$x = 2\left[1 + \left(1 + 4\beta\frac{G_0}{c_0^2 T}\ln\frac{r}{R_0}\right)^{1/2}\right]^{-1},$$
$$\theta \approx \frac{2\beta G_0}{c_0^2}\ln\frac{r}{R_0}. \tag{7}$$

G_0 is the maximum value of the function $G(\tau)$ on the surface of the sphere, x is a quantity characterizing the attenuation of the shock wave, R_0 and T are characteristic values of the channel radius and its expansion time.

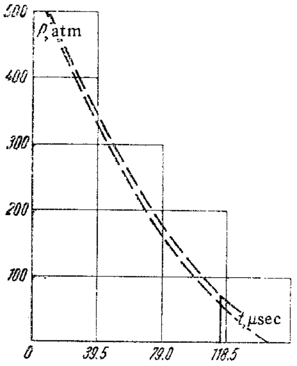

Fig. 1

Equation (7) are applicable if the following conditions are satisfied:

$$2\beta G_0/Tc_0^2 > 1,\ \ \ln(r/R_0) \gg 1. \tag{8}$$

The first of these conditions expresses the condition for formation of a discontinuity close to the discharge, while the second makes it possible to employ the asymptotic formulas of the Kirkwood-Bethe theory. Clearly, at large distances the shock wave parameters are determined simply by the maximum value of the function G_0, if the characteristic values, R_0 and T, are known.

In turn, for rough estimates, on the basis of Eq. (3) it is possible to assume that $G_0 \approx {}^3/_2 \, R^3_0 T^{-1}$, and the quantity R_0 can be found from [2]

$$R^5_0 = 3(\gamma - 1)\, T^2 E \,/\, 4\pi\rho_0\,, \tag{9}$$

where ρ_0 is the equilibrium density of the liquid.

Thus, it is possible to find the order of magnitude for the pressure in the shock wave radiated by an intense discharge from T the given duration of the discharge and E the total energy released in the channel

Using the above estimate for $G_0 \approx 3R^3_0 / 2T^2$ and Eq. (9), we can rewrite the first inequality of (8) as follows:

$$\frac{2\beta\, G_0}{c_0^2 T} = 6\left(\frac{R_0}{c_0 T}\right)^3 = \left(\frac{E/R_0^3}{\rho_0 c_0^2}\right)^{3/5} \approx \left(\frac{E}{\rho_0 c_0^5 T^3}\right)^{3/5} > 1\,. \tag{10}$$

Hence it follows that the possibility for formation of a shock wave depends on the ratio of the energy density in the channel (proportional to the channel pressure) to the characteristic pressure in the liquid $\rho_0 c^2{}_0$.

If the rate of expansion for the channel is less than the speed of sound, which is the case when instead of the first inequality of (8) the opposite inequality

$$2\beta G_0 \,/\, c^2_0 T < 1 \tag{11}$$

is satisfied, then shock waves may be formed at a certain distance from the discharge as a result of accumulating nonlinear effects. Taking this into account, in the approximation of nonlinear acoustics [6], we are led to the following expression for G_m

$$\frac{\beta G_m^2}{c_0^2} = \frac{5\, G_0 T}{4}\left[\ln\frac{r}{R_0}\right]^{-1}, \quad p_m = \frac{\rho_0 G_m}{r} \quad \text{at} \quad \ln\frac{r}{R_0} \geqslant \frac{c_0 T}{2\beta G_0} \tag{12}$$

If this condition is satisfied the shock wave is formed at a distance r.

Going over to numerical integration of (2)–(4), we note that on the basis of the experimental data (see, for example, [1] for the time dependence of the channel-power input we can take the "triangular" approximation

$$N(\tau) = \begin{cases} k\tau & (0 \leqslant \tau \leqslant {}^1/_2\, T) \\ -k\tau + kT & ({}^1/_2\, T \leqslant \tau \leqslant T) \\ 0 & (T < \tau)\,. \end{cases} \tag{13}$$

Here, T is the duration of the discharge (for a periodic discharge it may be assumed that each period has its own "triangle" in the power graph).

It is convenient to go over to the dimensionless variables

$$x = \frac{t}{T}\,, \quad y = \frac{R}{R_0}\,, \quad z = \frac{c}{c_0}\,, \quad \beta = \frac{pT^2}{\rho_0 R_0^2}, \qquad \eta = \frac{HT^2}{R_0^2}\,, \quad M = \frac{R_0}{c_0 T}$$

where T is the duration of the discharge, and R_0 is given by Eq. (9).

In the new variables (2)–(4) take the form:

$$\frac{d\beta}{dx} = 4\frac{\nu(x)}{y^3} - 3\gamma\,\frac{\beta}{y}\frac{dy}{dx}, \qquad \frac{d^2y}{dx^2} = \frac{1 + My'/z}{1 - My'/z}\,\frac{\eta}{y} + M\,\frac{1}{z}\frac{d\eta}{dx} - \frac{3}{2}\,\frac{1 - My'/3z}{1 - My'/z}\,\frac{y'^2}{y}\,,$$
$$\eta = \frac{1}{M^2(n-1)}\left[\left(nM^2\beta + \frac{B}{A}\right)^{(n-1)/n} - 1\right]. \tag{14}$$

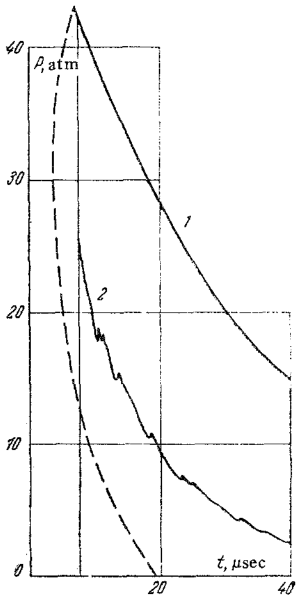

Fig. 2

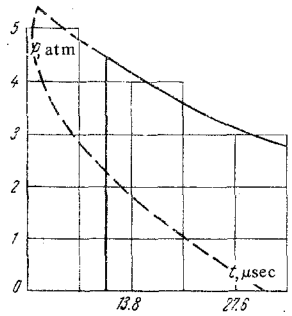

Fig. 3

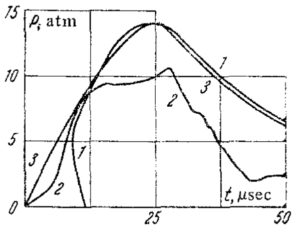

Fig. 4

Here $\nu(x)$ is the dimensionless power:

$$\nu(x) = \begin{cases} x & (0 \leqslant x \leqslant 1/2) \\ 1 - x & (1/2 \leqslant x \leqslant 1) \\ 0 & (1 < x) \end{cases}.$$

Equations (14) were integrated numerically on a computer with the initial conditions $y(0) = y_0$ and $y^{\cdot}(0) = 0$, after which the relations for $\rho(t)$ at various distances were found from Eqs. (5) and (6).

The position of the shock front and the amplitude of the shock wave were determined graphically from the "equal area" rule.

The calculated pressure profiles in the compression wave are shown in Figs. 1–4 (curve 1), which also give the available experimental results [2, 8] (curve 2).

Figure 1 shows the pressure profile at a point 1 m from the discharge gap for a discharge with parameters: energy released $E = 3 \cdot 10^4$ J, duration of discharge $T = 9$ μsec.

Figure 2 and 3 show the relation for p(t) at points 1 and 10 m, respectively, from a discharge with the parameters $E = 2.5 \cdot 10^3$ J and $T = 40$ μsec [8]; in constructing the graphs in Fig. 2 the calculated and shock fronts determined experimentally were combined.

Figure 4 shows the compression wave profile at a distance of .5m from the discharge with the parameters $E = 1020$ J and $T = 50$ μsec investigated in [2]. As distinct from the previous two discharges, in this case almost no shock waves are formed. Curve 3 in Fig. 4 represents the relation for p(t) obtained in the approximation of linear acoustics.

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